Lecture 8. Operational Semantics III
Functions & Recursions

$$\text{Fun } \overset{\downarrow}{X} \rightarrow \underset{=}{X} \quad \leftarrow \text{Bound var}$$

$$\underset{\triangle}{\text{def}} \qquad \text{use}$$

$$\leftarrow \text{Free var}$$

$$\text{Fun } \overset{\frown}{X} \rightarrow X + y$$

$$\text{def} \qquad \text{use}$$

$$e \equiv \text{Let } p = q + p \text{ In } (\text{Fun } q \rightarrow q + p) \; 3$$

$$e \; [10 \big/ q]$$

$$= \text{Let } p = 10 + p \text{ In } (\text{Fun } \overset{\downarrow}{q} \rightarrow q + p) \; 3$$

$$\frac{e_1 \Rightarrow v_1 \quad e_2 \Rightarrow v_2 \quad v_1, v_2 \in \mathbb{Z}}{e_1 + e_2 \Rightarrow v_1 + v_2 \in \mathbb{Z}}$$

Applications :   func↓ e_1 e_2 ↖ arg

Abstraction / Lambda : $\lambda x . e$

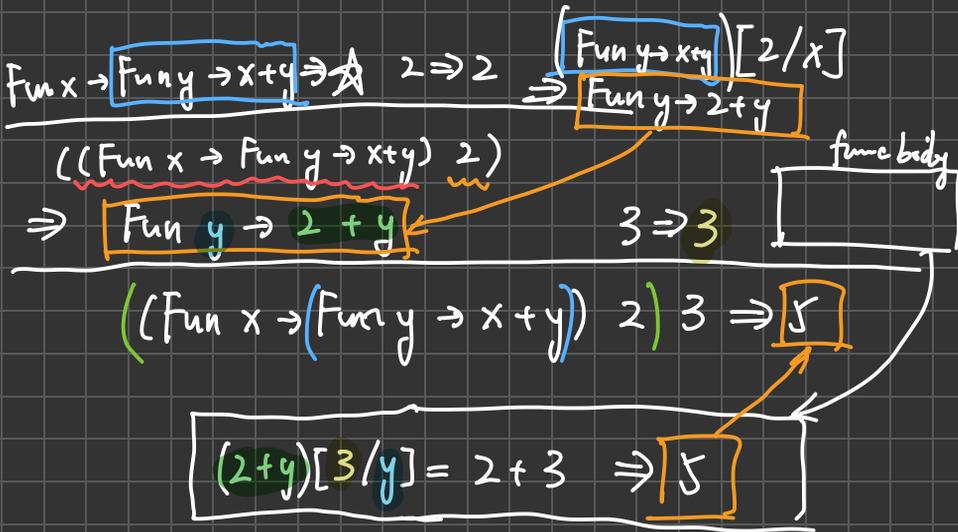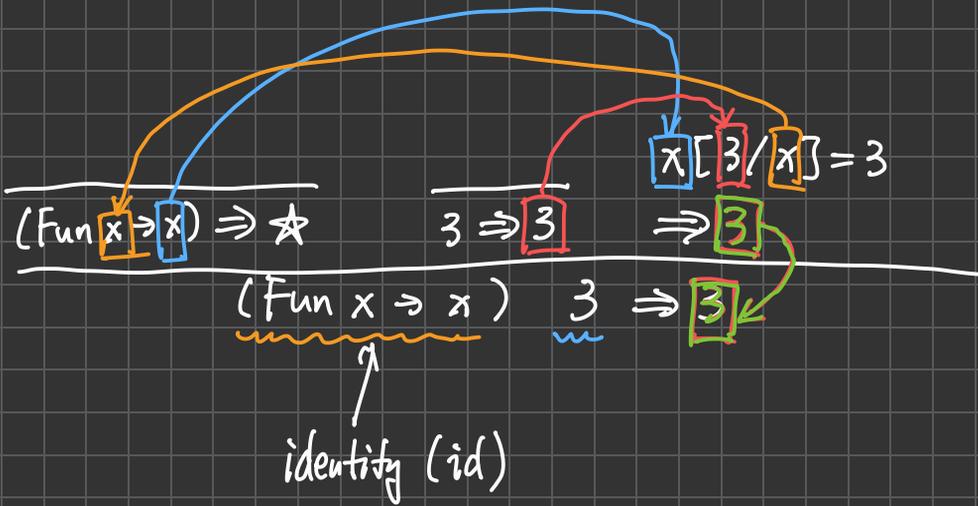$v ::= b \mid i \mid x \mid$ Fun $x \Rightarrow e$

Value
$(\text{Fun } x \Rightarrow x + 3) \Rightarrow$ Value $(\text{Fun } x \Rightarrow x + 3)$

[Value] $\dfrac{}{v \Rightarrow v}$

$(\text{Fun } x \Rightarrow x + 3) \Rightarrow \star$

Call-by-Name (CBN)   contrast   Call-by-value (CBV)
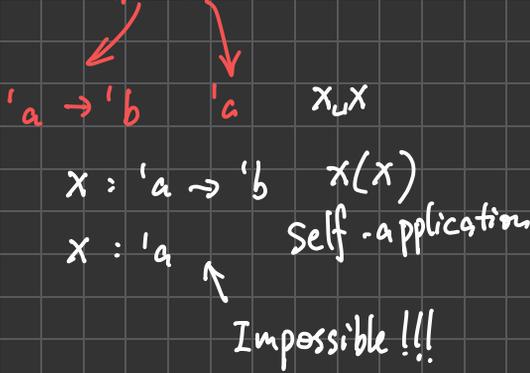
[App] $\dfrac{e_1 \Rightarrow \text{Fun } x \Rightarrow e' \qquad \boxed{e_2 \Rightarrow v_2} \qquad e'[v_2/x] \Rightarrow v}{e_1 \; e_2 \Rightarrow v}$

(Py)   reverse([1, 2] + [3, 4])

reverse([1, 2, 3, 4])

[4, 3, 2, 1]

$(Fun\ x \Rightarrow x) \Rightarrow \star$　　　$3 \Rightarrow 3$　　　$x[3/x] = 3$

$\Rightarrow 3$

$(Fun\ x \Rightarrow x)$　　$3 \Rightarrow 3$

identity (id)

$Fun\ x \rightarrow Fun\ y \rightarrow x+y \Rightarrow \star$　　$2 \Rightarrow 2$　　$(Fun\ y \rightarrow x+y)[2/x]$

$\Rightarrow Fun\ y \rightarrow 2+y$

$((Fun\ x \rightarrow Fun\ y \rightarrow x+y)\ 2)$

func body

$\Rightarrow Fun\ y \rightarrow 2+y$　　　　$3 \Rightarrow 3$

$((Fun\ x \rightarrow (Fun\ y \rightarrow x+y))\ 2)\ 3 \Rightarrow 5$

$(2+y)[3/y] = 2+3 \Rightarrow 5$

??

$(\text{Fun } x \to \boxed{x \ x})$ $(\text{Fun } x \to x \ x) \Rightarrow$??

$'a \to 'b \quad 'a \qquad x_,x$

$x : 'a \to 'b \qquad x(x)$

$x : 'a \qquad \qquad \text{Self-application}$

$\uparrow$

Impossible !!!

$\Omega : \text{diverge}$

$(x \ x)[\ M \ /x]$

$$\frac{M \Rightarrow \text{Fun } x \to x \ x \qquad M \Rightarrow M}{\Omega \equiv \boxed{M \ M} \Rightarrow ??} \qquad = M \ M$$

Diverges

Does not Normalize

$M \equiv \text{Fun } x \to x \ x$