$$e ::= \ \ldots$$
$$\mid \ \mathrm{Let}\ \boxed{\mathrm{Rec}}\ f\ x = e_1\ \mathrm{In}\ e_2$$

$$\underset{'a \to 'b}{e}\ \ \underset{'a}{e} \qquad \text{(Self-app)}$$

$$fact \overset{def}{=} \lambda n.\ n \leq 1\ ?\ 1 : fact(n-1) * n$$

$$fact \overset{def}{=} \lambda f.\ \lambda n.\ n \leq 1\ ?\ 1 : f\,(n-1) * n$$

$$F(f) = \lambda n.\ n \leq 1\ ?\ 1 : f\,(n-1) * n$$

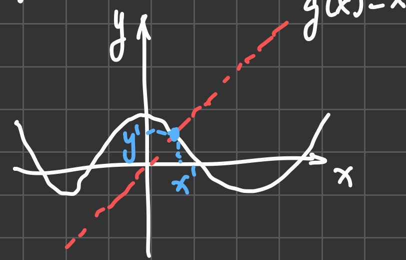
$$factorial : \mathbb{N} \to \mathbb{N}$$
$$f : \mathbb{N} \to \mathbb{N}$$
$$F(f) : \mathbb{N} \to \mathbb{N}$$

$$factorial = F(f) = f$$

Fix-point

if $f(x) = x$
then x is a fixpoint of f

$f(x) = \cos x$



x' is a fix point of f

$\text{Fixpoint}(f) = x$

$F(f) = \lambda n.\ n \le 1\ ?\ 1 : f(n-1) * n$

$F(f) = f$

$(\quad \dots \quad) f = f$

(Hypo) $\text{Fix} : (A \to A) \to A$

$\text{recursive}f = \text{Fix}\ (\lambda f.$

$\lambda\ \text{input}. \quad \text{body} \dots (\text{input}, f)$

$)$

$$e ::= \text{Let Rec } f\ x = e_1 \text{ In } e_2 \mid \ldots \quad (F^b)$$

$$[\text{Let Rec}]\ \frac{e_2[(\text{fun } x \to e_1[\,\text{Fun } x \to \text{Let Rec } f\ x = e_1 \text{ In } f\ x\,/f])/f] \Rightarrow v_2}{\text{Let Rec } \underset{\text{string / func name}}{f}\ \underset{\text{string var name}}{x} = \underset{\text{func body}}{e_1} \text{ In } \underset{\text{Continuation}}{e_2} \Rightarrow v_2}$$

$$[\text{Let}]\ \frac{e_1 \Rightarrow v_1 \qquad e_2[v_1/x] \Rightarrow v_2}{\text{Let } x = e_1 \text{ In } e_2 \Rightarrow v_2}$$

$$\left[\begin{array}{l} \text{Let Rec odd } x = \\ \quad \left\{\begin{array}{l} \text{If } \boxed{x} = 1 \text{ Then True} \\ \text{Else If } \boxed{x} = 0 \text{ Then False} \\ \quad \text{Else odd } (\boxed{x} - 2) \end{array}\right. \\ \text{In} \\ \text{Odd } 2 \end{array}\right] \Rightarrow \text{False}$$

$$\text{Odd } 2 \to \text{Odd } 0 \to \text{False}$$

$$\equiv \text{False}$$

$$\dfrac{\dfrac{\dfrac{}{e_1[\,0\,/x] \Rightarrow \text{False}}}{\vdots}}{(\text{odd } 0)[\;\cdots\;/\text{odd}]}$$

$$\dfrac{(\text{Fun } x \to \cdots) \Rightarrow ☆ \qquad 2 \Rightarrow 2 \qquad e_1[\;\cdots\; \text{In odd } x/\text{odd}][2/x] \Rightarrow \text{False}}{}$$

$$\text{Fun } x \to \text{Let Rec odd } x = e_1$$

$$(\text{Fun } x \to e_1 [\text{ In odd } x \qquad /\text{odd}])\; 2 \Rightarrow \text{False}$$

$$\equiv$$

$$\text{Fun } x \to \text{Let Rec odd } x = e_1$$

$$\dfrac{(\text{odd } 2)[\text{Fun } x \to e_1[\text{ In odd } x \;/\text{odd}]/\text{odd}] \Rightarrow \text{False}}{\text{Let Rec odd } x = e_1 \text{ In odd } 2 \Rightarrow \text{False}}$$

Mutual Recursion

```
function f() {
    g();
}
function g() {
    f();
}
```

cannot have f — can have f

let $f\ x = e_1$ In e_2

Let Rec $f\ x = e_1$ In e_2

can have f

Let Rec $f\ x = e_1$ In
    Let Rec $g\ y = e_2$ In
        e_3

Ocaml
Let rec $f\ x = e_1$
$\boxed{\text{And}}$ $g\ y = e_2$
In e_3

Record : $\{$

f : fun $x \to e_1$

g : fun $y \to e_2$

$\vdots$

$\}$

can contain f and g