Quiz 5: Encodings and Rewrite Rules

EN.601.426/626 Principles of Programming Languages – SP26

Name: _____

We have a few rewrite-rules for λ-calculus:

$$\begin{array}{lrll} (\alpha) & \lambda x.e & \equiv_\alpha & \lambda y.e[y/x] \quad (\text{with } y \notin FV(e)) \\ (\beta) & (\lambda x.e_1)\ e_2 & \to_\beta & e_1[e_2/x] \\ (\eta) & \lambda x.f\ x & \to_\eta & f \quad (\text{with } x \notin FV(f)) \end{array}$$

Perform the rewrites by following the rules:

$$\begin{array}{rl} & (\lambda x.x)\ ((\lambda y.y)\ z) \\ \to_\beta & \\ \to_\beta & \end{array}$$

$$\begin{array}{rl} & (\lambda x.\lambda y.x\ y)\ y \\ \equiv_\alpha & \\ \to_\beta & \end{array}$$

$$\begin{array}{rl} & \lambda x.((\lambda y.f\ y)\ x) \\ \to_\beta & \\ \to_\eta & \end{array}$$

Given the following definitions, perform the rewrites:

$$\begin{array}{rlrl} \text{TRUE} & \equiv_\delta\ \lambda x.\lambda y.x & \text{PAIR} & \equiv_\delta\ \lambda x.\lambda y.\lambda f.f\ x\ y \\ \text{FALSE} & \equiv_\delta\ \lambda x.\lambda y.y & \text{LEFT} & \equiv_\delta\ \lambda p.p\ (\lambda x.\lambda y.x) \\ \text{IF} & \equiv_\delta\ \lambda b.\lambda t.\lambda f.b\ t\ f & \text{RIGHT} & \equiv_\delta\ \_\_\_\_\_ \end{array}$$

$$\begin{array}{rl} & (\lambda t.\text{IF TRUE (LEFT } t\text{) (RIGHT } t\text{))} (\text{PAIR } a\ b) \\ \equiv_\delta & \end{array}$$